A Quantum Theory of Speciation

Kerry M. Soileau

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Abstract

We propose a theoretical model of quantum speciation among elements of a finite dimensional Hilbert space. The potential for species diversity and the current environment are represented by linear operators satisfying a compatibility criterion. A method for calculating probabilities of production of individuals is defined.

1 Introduction

Let H be a Hilbert space H with inner product $\langle \cdot, \cdot \rangle$ and finite dimension n. We say that the ordered pair (A, B) is compatible if A and B are linear operators on H, B is Hermitian and the composition AB has all real eigenvalues and a unique largest eigenvalue. By C_H we mean the collection of compatible ordered pairs.

Fix $(E, S) \in C_H$. Let the environment be represented by E, and the species by S. The interaction of the species with the environment is represented by the linear operator $R \equiv ES$. Let the unit eigenvectors of S be denoted by $V_S = \{s_1, s_2, s_3, \cdots, s_n\}$. This set represents the "individuals" genetically possible for the species represented by S. Note that V_S forms an orthonormal basis for H. Let the unit eigenvectors of R be denoted by V_R, and let $r \in V_R$ be the eigenvector with largest eigenvalue. The probability of production of the individual s_i is defined to be $|\langle r, s_i \rangle|^2$ for each $i \in \{1, 2, 3, \cdots, n\}$.

2 Example

Take the Hilbert space to be $\mathbb{R}^8$ with the usual topology and inner product $\langle \cdot, \cdot \rangle$. Let the environment be represented by the matrix

$$E = \begin{pmatrix} 0.58 & 0.38 & -0.49 & 0.45 & -0.87 & 0.53 & 0.58 & 0.61 \\ 0.38 & 0.25 & -0.32 & 0.29 & -0.57 & 0.34 & 0.38 & 0.40 \\ -0.49 & -0.32 & 3.01 & -0.38 & -1.39 & -0.45 & -0.49 & -0.52 \\ 0.45 & 0.29 & -0.38 & 0.35 & -0.68 & 0.41 & 0.45 & 0.48 \\ -0.87 & -0.57 & -1.39 & -0.68 & 3.07 & -0.79 & -0.87 & -0.92 \\ 0.53 & 0.34 & -0.45 & 0.41 & -0.79 & 0.48 & 0.52 & 0.56 \\ 0.58 & 0.38 & -0.49 & 0.45 & -0.87 & 0.52 & 0.57 & 0.61 \\ 0.61 & 0.40 & -0.52 & 0.48 & -0.92 & 0.56 & 0.61 & 0.65 \end{pmatrix} \tag{2.1}$$

and let the species be represented by the matrix

$$S = \begin{pmatrix} 4.62 & -0.33 & -0.95 & -0.41 & -0.37 & -0.21 & -1.33 & 0.06 \\ -0.33 & 7.96 & -0.10 & -0.04 & -0.04 & -0.02 & -0.13 & 0.00 \\ -0.95 & -0.10 & 6.10 & 0.43 & 0.78 & -0.28 & -0.72 & 0.04 \\ -0.41 & -0.04 & 0.43 & 5.32 & 0.12 & -0.11 & -0.91 & -0.16 \\ -0.37 & -0.04 & 0.78 & 0.12 & 2.96 & 0.24 & 0.61 & 0.08 \\ -0.21 & -0.02 & -0.28 & -0.11 & 0.24 & 3.62 & -0.01 & 0.93 \\ -1.33 & -0.13 & -0.72 & -0.91 & 0.61 & -0.01 & 4.03 & -0.12 \\ 0.06 & 0.00 & 0.04 & -0.16 & 0.08 & 0.93 & -0.12 & 1.35 \end{pmatrix} \tag{2.2}$$

The interaction of the species with the environment is given by the matrix

$$R = ES = \begin{pmatrix} 2.31 & 2.81 & -4.60 & 1.14 & -2.60 & 2.23 & 0.85 & 1.12 \\ 1.52 & 1.85 & -3.01 & 0.72 & -1.71 & 1.44 & 0.56 & 0.73 \\ -3.63 & -2.54 & 18.07 & -0.10 & -2.07 & -3.13 & -3.88 & -1.02 \\ 1.80 & 2.14 & -3.57 & 0.89 & -2.03 & 1.73 & 0.66 & 0.88 \\ -2.10 & -4.08 & -4.68 & -2.44 & 7.47 & -2.31 & 1.34 & -1.63 \\ 2.13 & 2.51 & -4.21 & 1.05 & -2.37 & 2.03 & 0.74 & 1.03 \\ 2.33 & 2.81 & -4.59 & 1.15 & -2.61 & 2.19 & 0.81 & 1.11 \\ 2.43 & 2.96 & -4.87 & 1.23 & -2.75 & 2.36 & 0.89 & 1.19 \end{pmatrix} \tag{2.3}$$

The eigenvectors of R are given by

$$V_R = \left\{ \begin{array}{c} (-0.26, -0.17, 0.81, -0.20, -0.09, -0.24, -0.26, -0.27), \\ (-0.25, -0.16, -0.28, -0.19, 0.78, -0.23, -0.25, -0.26), \\ (0.09, 0.60, 0.04, -0.39, -0.04, -0.54, 0.27, -0.34), \\ (-0.25, -0.08, 0.17, -0.18, -0.08, 0.45, 0.81, -0.07), \\ (-0.34, 0.30, -0.01, 0.38, 0.20, 0.40, -0.17, -0.65), \\ (0.59, 0.18, 0.31, -0.07, 0.58, 0.38, 0.09, 0.18), \\ (0.08, -0.05, -0.02, -0.23, -0.13, 0.40, -0.16, -0.86), \\ (-0.43, 0.12, -0.13, -0.39, -0.01, 0.09, -0.49, 0.62) \end{array} \right\} \quad (2.4)$$

The eigenvalues of the matrix R are given by

{22.53,12.08,0.06,-0.05,-0.03,0.02,0.01,0.00}.

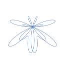
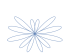
The eigenvector of R with largest eigenvalue is

(-0.26,-0.17,0.81,-0.20,-0.09,-0.24,-0.26,-0.27).

The eigenvectors of the matrix S are given by

$$
V_S = \left\{ \begin{array}{c}
(0.10, -1.00, 0.01, 0.00, 0.00, 0.00, 0.00, 0.00), \\
(0.35, 0.03, -0.83, -0.38, -0.18, 0.05, 0.12, 0.01), \\
(-0.56, -0.05, 0.04, -0.48, 0.20, 0.09, 0.64, 0.01), \\
(-0.40, -0.04, -0.48, 0.74, 0.01, 0.08, 0.22, -0.03), \\
(0.02, 0.00, 0.02, -0.01, 0.14, 0.92, -0.17, 0.34), \\
(-0.49, -0.05, 0.00, -0.17, -0.78, 0.06, -0.33, 0.02), \\
(0.39, 0.04, 0.29, 0.21, -0.55, 0.17, 0.62, 0.06), \\
(-0.03, 0.00, -0.03, 0.03, 0.00, -0.34, 0.02, 0.94),
\end{array} \right\} \tag{2.5}
$$

The probabilities of production are as shown in the following table:

	Individual	Probability
	(-0.26, -0.17, 0.81, -0.20, -0.09, -0.24, -0.26, -0.27)	0.02
	(-0.25, -0.16, -0.28, -0.19, 0.78, -0.23, -0.25, -0.26)	0.52
	(0.09, 0.60, 0.04, -0.39, -0.04, -0.54, 0.27, -0.34)	0.01
	(-0.25, -0.08, 0.17, -0.18, -0.08, 0.45, 0.81, -0.07)	0.24
	(-0.34, 0.30, -0.01, 0.38, 0.20, 0.40, -0.17, -0.65)	0.07
	(0.59, 0.18, 0.31, -0.07, 0.58, 0.38, 0.09, 0.18)	0.09
	(0.08, -0.05, -0.02, -0.23, -0.13, 0.40, -0.16, -0.86)	0.01
	(-0.43, 0.12, -0.13, -0.39, -0.01, 0.09, -0.49, 0.62)	0.04

3 Motivation

In quantum mechanics, observables are represented by self-adjoint operators on a Hilbert space. Thus in proposing a model of quantum speciation, it is natural to regard a species as a whole as some self-adjoint operator S. In the quantum mechanical setting, each possible measurement of an observable corresponds to a unit eigenvector and eigenvalue of this operator, so by analogy we regard each unit eigenvector of the species linear operator S to represent a possible individual. We postulate that each species will have only finitely many possible individuals, thus we assume that S, and also the Hilbert space, have finite dimension. Thus S is in fact Hermitian. We may then regard the eigenvalues of each unit eigenvector (i.e. individual) of S as representing the reproductive strength of that individual. We model the influence of the environment by means of a linear operator E which is composed with S to produce the resultant operator R. We require that (E, S) be compatible, in the sense defined above, so that R will have all real eigenvalues and a unique largest eigenvalue.

The definition of probability of production was motivated by the following observation. If $\vec{\phi}$ is a random vector in $\mathbb{R}^n$, how may we determine the unit vector $\hat{v} \in \mathbb{R}^n$ which maximizes the expectation value $E\left(\vec{\phi} \cdot \hat{v}\right)^2$? It is not difficult to show that this is accomplished by taking $\hat{v}$ to be the eigenvector

$\hat{v}_{\max}$ with maximal eigenvalue of the matrix

$$\begin{pmatrix} E\phi_1\phi_1 & E\phi_1\phi_2 & \cdots & E\phi_1\phi_n \\ E\phi_2\phi_1 & E\phi_2\phi_2 & \cdots & E\phi_2\phi_n \\ \vdots & \vdots & \ddots & \vdots \\ E\phi_n\phi_1 & E\phi_n\phi_2 & \cdots & E\phi_n\phi_n \end{pmatrix}.$$

The maximal value of $E\left(\vec{\phi}\cdot\hat{v}\right)^2$ is then equal to this eigenvalue. We may express $\hat{v}_{\max}$ as a unique linear combination of the eigenvectors (individuals) of S, like so: $\hat{v}_{\max} = \sum_{i=1}^{N} (\hat{v}_{\max}\cdot\hat{s}_i)\,\hat{s}_i$. Again following the pattern seen in the quantum mechanical setting, we define the probability of "observing," i.e. producing the individual represented by $\hat{s}_i$ as $|\langle\hat{v}_{\max},\hat{s}_i\rangle|^2 = (\hat{v}_{\max}\cdot\hat{s}_i)^2$. Although the motivation involves Hermitian operators R, this is not assumed in the definition of compatible operators.